

Quantum Entanglement and Teleportation in Higher Dimensional Black Hole Spacetimes

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We study the properties of quantum entanglement and teleportation in the background of stationary and rotating curved space-times with extra dimensions. We show that a maximally entangled Bell state in an inertial frame becomes less entangled in curved space due to the well-known Hawking-Unruh effect. The degree of entanglement is found to be degraded with increasing the extra dimensions. For a finite black hole surface gravity, the observer may choose higher frequency mode to keep high level entanglement. The fidelity of quantum teleportation is also reduced because of the Hawking-Unruh effect. We discuss the fidelity as a function of extra dimensions, mode frequency, black hole mass and black hole angular momentum parameter for both bosonic and fermionic resources.

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I. INTRODUCTION

The new field of quantum information has made rapid progresses in recent years. As relativistic field theory provides not only a more complete theoretical framework but also many experimental setups, relativistic quantum information theory may become an essential theory in the near future, with possible applications to quantum teleportation. Quantum entanglement has already been studied in relativistic frames, inertial or not [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Czachor [2] studied a version in which an electron is in paramagnetic resonance with relativistic particles, and Peres et al demonstrated that the spin of an electron is not covariant under Lorentz transformation [3]. Moreover, Alsing and Milburn [4] studied the effect of Lorentz transformation on maximally spin-entangled Bell states in momentum eigenstates and Gingrich and Adami [5] derived a general transformation rule for the spin-momentum entanglement of two qubits. The recent work of Alsing and Milburn extended the result to a situation where one observer is accelerated [6]. Fuentes-Schuller and Mann calculated the entanglement between two free modes as seen by an inertial observer detecting one mode and a uniformly accelerated observer detecting the other mode [9]. Recently, one of us (X.-H.G.) discussed a possible extension to the gravitational field of the quantum teleportation in four dimensional spacetime [10, 11].

Quantum information theory has not generally been regarded as a theory of spacetime, and entanglement and teleportation has usually been discussed only in four dimensional spacetime. However, if our spacetime is not the usual four dimensional one, the structure of spacetime, in particular, the extra dimensions may still influence the nature of entanglement. In the last decade, string theory with an extra space compactified at a larger length scale or lower energy scale than the Planck scale has been an attractive idea to solve the gauge hierarchy problem and possibly a candidate for quantum gravity[12]. The interest on extra dimensions has escalated from an expectation that in the coming years Large Hardon Collider (LHC) at CERN may create mini-black holes and thus signal the effects of extra dimensions because of the relevant energy scale[13]. Even if mini-black holes may be created at LHC, probing the extra dimensions will require a very complicated and delicate analysis of cornucopia of produced particles and their interactions.

As far as extra dimensions are concerned, it would be interesting to think about alternative methods to probe them. As an alternative, we wondered how extra dimensions might influence, for instance, quantum teleportation by using some newly developed technology. In this paper, we investigate quantum entanglement and teleportation in a more general situation – in the spacetimes of higher dimensional black holes. We will show how the extra dimensions and

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even TeV-level gravity would change the properties of entanglement and teleportation, when two observers stay on a 3-brane (our universe), but not in the bulk. Our scheme differs from the standard teleportation protocol in that one observer, Alice, stays stationary at an asymptotically flat region of a black hole, while the other observer, Bob, moves from the Alice's place toward the black hole. We assume that both Alice and Bob hold an optical cavity for the measurements of the inside state. They instantaneously share an entangled Bell state at the same initial point in the flat region. Then Bob picks his qubit and travels to the event horizon of the black hole. Teleportation can be performed between Alice and Bob when Bob stays at a fixed radius near the event horizon.

However, in contrast with the Minkowski spacetime, quantum teleportation in a black hole spacetime requires several points to be clarified such as Bob's trajectory and the shape of the cavity in which particles are confined. It is well known that entanglement states are sensitive to the environment and Bob's trajectory may influence his description of the qubit state. For instance, if Bob's cavity is not perfectly insulated, thermal particles from the black hole might flow into the cavity. Fortunately, if Bob approaches the black hole by freely falling, this situation can be avoided, since a geodesic detector sees no particle coming out of the black hole [14]. Thus, after sharing the initial entangled state, Bob may freely fall and then stop freely falling and become stationary at a fixed radius, but to avoid the infinite deceleration due to an instantaneous stop he may slowly decelerate for a while and then become stationary at the fixed radius. During this process the cavity might be teeming with thermal particles, which does not matter seriously, because what is needed is the degree of entanglement and teleportation under this condition.

On the other hand, if the cavity is totally reflecting and does not couple at all to the outside, one can move Bob's cavity (a rectangular box) adiabatically to near the black hole horizon, which does not introduce any thermal particle of the Unruh vacuum inside the cavity. The Boulware vacuum is the ground state for the inside of a box that is stationary in a stationary black hole whose timelike Killing vector is used to define the ground state, while the Unruh vacuum is the Minkowski vacuum that has thermal particles relative to the Boulware vacuum and can be detected by an accelerated observer [15]. Unruh and Wald discussed about mining energy from a black hole by lowering an originally empty box to a black hole sufficiently slowly, opening up the box and letting the surrounding Unruh thermal radiation flow into the box [16]. They also found that during this process, if one does not open the box, then the interior of the box will remain in the vacuum state with respect to its local time coordinate. The reason is that the boundaries of the box that both move with an acceleration effectively remove the horizon from consideration and thereby keep the field inside the closed box in the vacuum state.

Of course, even for a well-insulated and reflecting cavity, one cannot screen gravity and an initial vacuum state inside the cavity might evolve into thermal Unruh states because of gravitational coupling between the inside of the cavity and the outside. We estimate a possible coupling time for a Boulware vacuum inside a cavity, which perfectly reflects non-gravitational fields, to become approximately thermal due to gravitational effects. For a thick wall cavity outside a solar mass black hole, the coupling time is larger than the age of our universe, which implies that the gravitational coupling effect is negligible. Note that this works under the condition that the size of the cavity is much smaller compared to the curvature of the black hole. Otherwise, the cavity cannot act as it were in a thermal bath that is equilibrium with the black hole. For a mini-black hole that may be created at LHC with typical horizon curvature 10^{-19}m , it is hard to make a thick wall box (say, of the size $1\text{m} \times 1\text{m} \times 1\text{m}$) near the horizon in thermal equilibrium. And also, the coupling between the inside and the outside will diverge if one takes an infinitely thin box and couples the inside with the outside across a tiny distance.

Now our situation is that what we put into Bob's cavity initially is not a pure state but a state entangled with a particle inside Alice's cavity. This entanglement will be maintained by any evolution of the cavity as long as the cavity remains totally reflecting and decoupled from any outside system. In this case, the entanglement between Bob's particle and Alice's particle will survive and perfect teleportation is still possible. This process works well for teleportation outside massive black holes, but for mini-black holes, especially those short-lived black holes that may be created at LHC, it is impossible to lower Bob's cavity slowly to approach black hole horizons.

The main purpose of this paper is to investigate possible influences that extra dimensions might impose on the entanglement and teleportation in curved spacetimes. So we do not discuss the situation, in which both cavities evolve adiabatically, because this makes no distinctions between teleportation in a curved spacetime and that in a flat spacetime. Contrarily, we consider the situation, in which Bob freely falls, then slow downs for a finite period of time and stops freely falling and becomes stationary at a fixed radius. Then, measurements of teleportation can be performed between Alice and Bob. The results show that in a higher dimensional black hole spacetime the degree of entanglement and the fidelity of teleportation is closely related to the extra dimensions, the mode frequency, the mass and the angular momentum per unit mass of the black hole.

The organization of this paper is as follows. In Sec. II, we study quantum entanglement and teleportation in a higher dimensional Schwarzschild black hole spacetime. After reviewing a basic feature of quantum field theory in this curved spacetime, we estimate the gravitational coupling time for states inside and outside the cavity and then go directly to the discussion of entanglement and teleportation. In Sec. III, we extend our discussion to a rotating black hole spacetime. The discussion applies not only to scalar particles but also to gauge bosons and Dirac

particles. Finally, we discuss the effects of the extra dimensions, mode-frequency and black hole parameters on fidelity of quantum teleportation in Sec. IV.

II. HIGHER DIMENSIONAL SCHWARZSCHILD BLACK HOLE CASES

A. Hawking radiation and vacuum structure

We first review some essential feature of quantum field theory in a higher dimensional Schwarzschild black hole, which is relevant to quantum entanglement. It is the global structure of a black hole spacetime that plays an important role in understanding Hawking radiation and black hole thermodynamics [14, 17]. It has also been known that the vacuum of a nonstationary curved spacetime is in general not equivalent to the Minkowski vacuum. The event horizon of a black hole causally disconnects the exterior region from the interior one. The global spacetime structure of a uniformly accelerated observer consists of two Rindler wedges: one is causally disconnected from the other by the future and past horizons. Thus the accelerated observer detects a thermal spectrum while moving through the Minkowski vacuum, the so-called Unruh effect [14, 18]. The spacetime near the event horizon of a Schwarzschild black hole approximately looks like the Rindler spacetime, which provides the Hawking radiation with an interpretation of the Unruh effect. Observing that the causally disconnected region surrounded by horizons provides a fictitious system to an outside observer, Israel [19] applied thermofield dynamics [20] to the Rindler wedge or black holes to explain their thermal nature (see also [21]).

In the following, we use the two-photon state of the electromagnetic field, which may be modeled by a massless scalar field, where the polarization is ignored [6]. The metric of a d -dimensional Schwarzschild spacetime is given by [22]

$$ds^2 = - \left[1 - \left(\frac{r_h}{r} \right)^{d-3} \right] dt^2 + \left[1 - \left(\frac{r_h}{r} \right)^{d-3} \right]^{-1} dr^2 + r^2 d\Omega_{d-2}^2, \quad (1)$$

where r_h denotes the event horizon with the area $A_d = r_h^{d-2} \Omega_{d-2}$, where Ω_{d-2} is the volume of a unit $(d-2)$ -sphere. The mass of the d -dimensional black hole is given by

$$M = \frac{(d-2)r_h^{d-3}\Omega_{d-2}}{16\pi G_d} \quad (2)$$

for the d -dimensional Newton's constant G_d . The Klein-Gordon equation for the massless scalar field,

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0, \quad (3)$$

has the positive frequency mode solution

$$\phi(t, r, \theta, \varphi) = e^{-i\omega t} \frac{R_{\omega l}(r)}{r^{\frac{d-2}{2}}} Y_{lm}(\Omega), \quad (4)$$

where $Y_{lm}(\Omega)$ is the $(3+n)$ -spatial-dimensional generalization of the usual spherical harmonic functions depending on the angular coordinates, and $R_{\omega l}(r)$ satisfies the radial equation

$$\frac{\partial^2 R_{\omega l}}{\partial r_*^2} + \omega^2 R_{\omega l} - f(r) \left[\frac{(d-2)^2}{4r^2} f(r) + \frac{d-2}{2r} \frac{df}{dr} + \frac{l(l+n+1)}{r^2} \right] R_{\omega l} = 0. \quad (5)$$

Here $n = d - 4$ is the extra dimensions, and $f(r) = [1 - (r_h/r)^{d-3}]$ and $r_* = \int dr/f(r)$ denotes a tortoise coordinate, which may be exactly computed by expanding the denominator in partial fractions to yield the result [23]

$$r_* = r + \frac{r_h}{d-3} \sum_{k=0}^{d-4} \ln \left[\frac{r}{r_h} - e^{-i\frac{2\pi k}{d-3}} \right]. \quad (6)$$

To quantize the scalar field, we make use of the global structure of the spacetime given by the Penrose diagram for the extended Kruskal manifold in Fig. 1. The particle states are defined by positive frequency modes or wave packets

with respect to timelike Killing vectors. Note that the metric (1) has a future-directed Killing vector ∂_t in I and ∂_{-t} in II of Fig. 1. Solving (5), we obtain the positive frequency solutions

$$\phi_{I,p} = e^{-i\omega t} R_{\omega l} = e^{-i\omega(t+r_*)}, \quad (7)$$

$$\phi_{II,p} = e^{-i\omega t} R_{\omega l} = e^{-i\omega(t-r_*)}, \quad (8)$$

where p stands for (ω, l, m) . The wave packets may be found such that $\phi_{I,p}$ ($\phi_{II,p}$) have a support in I (II) but vanish in II (I), which may constitute a complete set along a Cauchy surface $t = 0$ [18]. Using (7) and (8), the quantum field may be quantized as

$$\phi = \sum_p [b_p^I \phi_{I,p} + b_p^{II} \phi_{II,p} + b_p^{I\dagger} \phi_{I,p}^* + b_p^{II\dagger} \phi_{II,p}^*] \quad (9)$$

where the operators b_p^I (b_p^{II}) and $b_p^{I\dagger}$ ($b_p^{II\dagger}$) are the annihilation and creation operators in region $I(II)$, respectively. The Fulling-Rindler vacuum is defined as

$$b_p^I |0\rangle_I \otimes |0\rangle_{II} = b_p^{II} |0\rangle_I \otimes |0\rangle_{II} = 0. \quad (10)$$

Rewriting (7) and (8) in Kruskal coordinates as

$$\phi_I = (-U/a)^{i\omega a}, \quad \phi_{II} = (U/a)^{-i\omega a}, \quad (11)$$

where $a = 1/\kappa = 2r_h/(d-3)$ for the surface gravity κ , (11) may be analytically continued in the lower half-plane of U as

$$\tilde{\phi}_{II} = e^{\pi\omega a} (U/a)^{i\omega a}. \quad (12)$$

On the other hand, the Schwarzschild metric in Kruskal coordinates takes the form

$$\begin{aligned} ds^2 &= - \left[1 - \left(\frac{r_h}{r} \right)^{d-3} \right] dU dV + r^2 d\Omega_{d-2}^2, \\ U &= \mp \frac{2r_h}{d-3} e^{-\frac{d-3}{2r_h} u}, \quad V = \pm \frac{2r_h}{d-3} e^{\frac{d-3}{2r_h} v}, \\ u &= t - r_*, \quad v = t + r_*, \end{aligned} \quad (13)$$

where the upper (lower) sign is for $I(II)$. Now $\partial/\partial U$ is another timelike Killing vector. The mode that is positive (negative) with respect to $\partial/\partial U$ and extends over the whole $V = 0$, may be found

$$\phi_1 = \phi_I + \tilde{\phi}_{II} = (-U/a)^{i\omega a} + e^{\pi\omega a} (U/a)^{i\omega a}, \quad (14)$$

$$\phi_2 = \phi_I^* + \tilde{\phi}_{II}^* = (-U/a)^{-i\omega a} + e^{-\pi\omega a} (U/a)^{-i\omega a}. \quad (15)$$

Note that the properly normalized mode $\phi_1/(e^{2\pi\omega a} - 1)^{1/2}$ leads to the spectrum of Hawking radiation. Using the normalized modes

$$\begin{aligned} \phi_+^* &= e^{-\pi\omega a/2} \phi_1 = e^{-\pi\omega a/2} (-U/a)^{i\omega a} + e^{\pi\omega a/2} (U/a)^{i\omega a}, \\ \phi_-^* &= e^{\pi\omega a/2} \phi_2 = e^{\pi\omega a/2} (-U/a)^{-i\omega a} + e^{-\pi\omega a/2} (U/a)^{-i\omega a}, \end{aligned} \quad (16)$$

we quantize the scalar field as

$$\phi = \sum_p [2 \sinh(\pi\omega a)]^{-1/2} [d_p^I \phi_+ + d_p^{II} \phi_- + d_p^{I\dagger} \phi_+^* + d_p^{II\dagger} \phi_-^*]. \quad (17)$$

The Kruskal operators are related with the Schwarzschild operators by the Bogoliubov transformations

$$\begin{aligned} b_p^I &= [2 \sinh(\pi\omega a)]^{-1/2} \left(e^{\pi\omega a/2} d_p^I + e^{-\pi\omega a/2} d_p^{II\dagger} \right), \\ b_p^{II} &= [2 \sinh(\pi\omega a)]^{-1/2} \left(e^{\pi\omega a/2} d_p^{II} + e^{-\pi\omega a/2} d_p^{I\dagger} \right). \end{aligned} \quad (18)$$

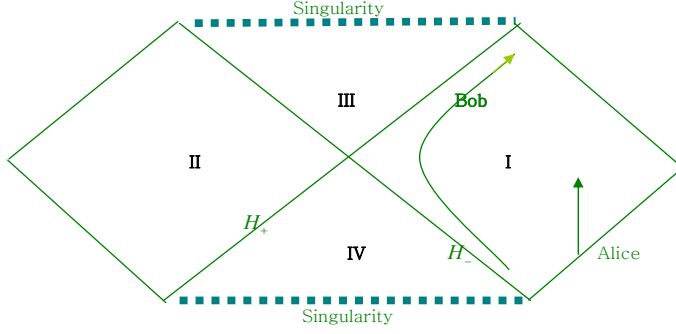


FIG. 1: Penrose diagram for the extended Kruskal manifold.

The Bogoliubov transformation for each mode may be expressed through a unitary transformation as

$$b_p^\sigma = S_p(r) d_p^\sigma S_p^\dagger(t), \quad (\sigma = I, II), \quad (19)$$

in terms of the two-mode squeeze operator

$$\begin{aligned} S_p(r) &= \exp[r(d_p^I d_p^{II} - d_p^{I\dagger} d_p^{II\dagger})] \\ &= \exp[-\tanh r (d_p^{I\dagger} d_p^{II\dagger})] \exp[-\ln \cosh r (d_p^{I\dagger} d_p^I + d_p^{II\dagger} d_p^{II} + 1)] \exp[\tanh r (d_p^I d_p^{II})], \end{aligned} \quad (20)$$

where $\tanh r = e^{-\pi\omega a}$. The inverse transformation is given by

$$d_p^\sigma = T_p(r)b_p^\sigma T_p^\dagger(t), \quad (\sigma = I, II), \quad (21)$$

where $T_p(r)$ is another two-mode squeeze operator

$$\begin{aligned} T_p(r) &= \exp[-r(b_p^I b_p^{II} - b_p^{I\dagger} b_p^{II\dagger})] \\ &= \exp[\tanh r (b_p^{I\dagger} b_p^{II\dagger})] \exp[-\ln \cosh r (b_p^{I\dagger} b_p^I + b_p^{II\dagger} b_p^{II} + 1)] \exp[-\tanh r (b_p^I b_p^{II})]. \end{aligned} \quad (22)$$

Now the Minkowski vacuum defined by

$$d_p^I |0\rangle_M = d_p^{II} |0\rangle_M = 0, \quad (23)$$

is simply given by

$$\begin{aligned} |0\rangle_M &= \prod_p T_p(r) |0\rangle_I \otimes |0\rangle_{II} \\ &= \prod_p \cosh^{-1} r \exp[\tanh r (b_p^{I\dagger} b_p^{II\dagger})] |0\rangle_I \otimes |0\rangle_{II} \\ &= \sum_{n_p=0}^{\infty} \prod_p \cosh^{-1} r \tanh^{n_p} r |n_p\rangle_I \otimes |n_p\rangle_{II}. \end{aligned} \quad (24)$$

where $|n_p\rangle_I = (b_p^{I\dagger})^n |0\rangle_I / \sqrt{n!}$ and $|n_p\rangle_{II} = (b_p^{II\dagger})^n |0\rangle_{II} / \sqrt{n!}$ are orthonormal bases for Hilbert space H_I and H_{II} , respectively. Equation (24) shows that the original vacuum state evolves into an Einstein-Podolsky-Rosen type correlation. Also note that the vacuum (24) is an extension to I and II of the thermal state in I with the inverse temperature $\beta = 2\pi a = 2\pi/\kappa$, which is the essence of thermofield dynamics [20]. Thus the vacuum (24) and its excited states are mixed states of outgoing particles in H_I . For an observer outside the black hole, quantum unitarity is lost because he would not be able to do any measurement in H_{II} . As H_{II} is no longer accessible to him, he can only make an average over the states in H_{II} to obtain the density operator in H_I . In fact, the measurement of an operator \mathcal{O}_I in I is the trace of \mathcal{O}_I weighted with the thermal operator $\rho_I = e^{-\beta b^{I\dagger} b^I} / Z_I$, ($Z_I = \text{Tr} \rho_I$).

One can simplify the analysis by considering the effect of teleportation of the target state $|\varphi\rangle_M = a|0\rangle_M + b|1\rangle_M$ by the Minkowski observer Alice to a single Schwarzschild mode of the observer Bob. Thus, one can only consider the mode p in region I which is distinct from the negative mode in the same region [6]. We emphasize that both Alice's and Bob's cavities are designed to detect particles, so we neglect the antiparticle modes in their cavities in the hereafter discussions. Therefore, the single-mode component of the Minkowski vacuum state, namely the two-mode squeezed state, is given by

$$|0\rangle_M^{\mathcal{B}} = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n\rangle_I \otimes |n\rangle_{II}, \quad (25)$$

and, similarly, the excited state by

$$\begin{aligned} |1\rangle_M^{\mathcal{B}} &= d_p^{I\dagger} |0\rangle_M \\ &= \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r \sqrt{n+1} |n+1\rangle_I \otimes |n\rangle_{II}. \end{aligned} \quad (26)$$

B. Entanglement and teleportation

We now discuss how Alice and Bob come to share an entangled resource for teleportation. After the coincidence of Alice and Bob, Alice remains at the asymptotical flat region, while Bob slowly lowers himself and his cavity to the vicinity of the black hole horizon. As noted previously, we assume the inside of Bob's cavity is totally decoupled from the outside. Thus, when the cavity is lowered to a fixed radius outside the horizon and becomes stationary, the entanglement between Bob's particle and Alice's particle will be exactly maintained. In this case, perfect entanglement and teleportation is possible and for massive black holes, this protocol works well.

However, a puzzle might be raised that nobody can screen gravity, so there would be some gravitational coupling between the inside and the outside of the cavity, even if the cavity is well-insulated and reflecting all non-gravitational

fields. Hence, the Boulware vacuum inside the cavity that is perfectly insulated from non-gravitational fields from the outside, might couple gravitationally to the outside and evolve into approximately the Unruh thermal state. We estimate the coupling time for the inside to become approximately thermal through the gravitational interaction. Let us consider a thick-wall box (for example, the thickness of the bottom wall is $l = 1\text{m}$), which is dropped very near the horizon of a Schwarzschild black hole, and the mass of the black hole is assumed to be of order of a solar mass ($M_{bh} \sim M_\odot$) and thus the black hole temperature is about $T_{bh} \sim 10^{-8} \text{ K}$. The size of the cavity must be much smaller than that of a black hole, otherwise it cannot maintain thermal equilibrium with the Hawking radiation.

Now no fields are confined to this rectangular box with an inside volume $V_c = 1\text{m} \times 1\text{m} \times 1\text{m}$. Then, the cavity outside the horizon will respond as though it were in thermal bath of temperature $T = T_{bh}/\chi$, where χ is the redshift factor defined by $\chi = \sqrt{-g_{00}}$. The energy density of thermal radiation is given by the Stefan-Boltzmann law

$$\rho = \alpha T^4, \quad \alpha = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} \quad (27)$$

We assume that after a time interval t , the box will be teeming with thermal flux. Hence, from an infinity observer viewpoint, a graviton with energy $h\nu_0$, crossing the thick walls of the cavity obey the following equation

$$h\nu \approx h\nu_0 - \frac{h\nu_0}{c^2} \kappa \Delta l, \quad (28)$$

where $\kappa = 2\pi c k_B T_{bh}/\hbar$ is the surface gravity of the black hole and $\Delta l = 1\text{m}$. We then have

$$\frac{\Delta \nu}{\nu_0} = \frac{\kappa}{c^2} \Delta l. \quad (29)$$

A clock inside the cavity will become slower since the cavity filled with a thermal radiation is heavier than before. Thus the change of the clock running from Eq. (29) is

$$\frac{\Delta t}{t} = \frac{\kappa}{c^2} \Delta l. \quad (30)$$

According to the uncertainty principle, the uncertainty of time is attributed to the change of energy inside the cavity, that is, $\Delta E \Delta t \sim \hbar$, where $\Delta E = \alpha T_{bh}^4 V_c$. Finally, we find that

$$t \sim \frac{\hbar}{\frac{\kappa}{c^2} \Delta l \Delta E}. \quad (31)$$

The results show that in our case $t \sim 10^{19} \text{ s}$, which is longer than the present age of our universe ($\approx 4 \times 10^{17} \text{ s}$). Therefore, the gravitational coupling between the inside and the outside of the cavity is negligible. For mini-black holes, the above estimation about gravitational coupling time does not work in that it is difficult to define a thick-wall cavity surrounded by a thermal bath near the black hole horizon. Therefore, for massive black holes, Bob can slowly lower his cavity down to the black hole horizon and the state inside the cavity will evolve adiabatically. Since gravitation coupling is negligible, perfect entanglement and teleportation is still possible for observers with a well insulated cavity outside a black hole. However, for mini-black holes, it is technically difficult to consider a very thin cavity to perform teleportation without losing information.

As emphasized in Sec. I, in this paper, we focus on the condition that Bob can freely fall toward the black hole and then stop through a slow acceleration on a surface outside the event horizon, and Bob's cavity evolves not totally adiabatically and becomes full of thermal radiation near the black hole horizon. Then the state inside Bob's cavity is no longer perfect entangled with that of Alice. One can assume that prior to their coincidence, Alice and Bob have no photons in their cavities. Suppose that each cavity supports two orthogonal modes, with the same frequency, labeled A_i and B_i with $i = 1, 2$, which are each excited to a single photon Fock state at the coincidence point. The state held by Alice and Bob is then the entangled Bell state

$$|\phi\rangle_M = \frac{1}{\sqrt{2}} (|0\rangle_M^{\mathcal{A}} |0\rangle_M^{\mathcal{B}} + |1\rangle_M^{\mathcal{A}} |1\rangle_M^{\mathcal{B}}), \quad (32)$$

where the first qubit in each term refers to Alice's cavity with index \mathcal{A} , the second qubit to Bob's cavity with index \mathcal{B} . The Bell state is a maximally entangled state in inertial frames. If Bob undergoes a uniform acceleration or stays in curved space-time, the state in his cavity must be specified in Rindler or Schwarzschild coordinates. As a consequence, the second state in each term of (32) should have a Schwarzschild mode expansion given by (25) and (26). We can then rewrite Eq. (32) in terms of Minkowski modes for Alice and Schwarzschild modes for Bob. Since

Bob is causally disconnected from region II , we must trace over the states in this region, which results in a mixed state [9]

$$\begin{aligned}\rho_{AB} &= \frac{1}{2 \cosh^2 r} \sum_n (\tanh r)^{2n} \rho_n, \\ \rho_n &= |0\rangle\langle 0, n| + \frac{\sqrt{n+1}}{\cosh r} |0, n\rangle\langle 1, n+1| + \frac{\sqrt{n+1}}{\cosh r} |1, n+1\rangle\langle 0, n| + \frac{n+1}{\cosh^2 r} |1, n+1\rangle\langle 1, n+1|,\end{aligned}\quad (33)$$

where $|n, m\rangle = |n\rangle_M^A |m\rangle_I^B$. The partial transpose of the density operator ρ_{AB} can be obtained by interchanging Alice's qubits as

$$\rho_{AB}^T = \frac{\tanh^{2n} r}{2 \cosh^2 r} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{n+1}}{\cosh r} & 0 \\ 0 & \frac{\sqrt{n+1}}{\cosh r} & 0 & 0 \\ 0 & 0 & 0 & \frac{n+1}{\cosh r} \end{pmatrix},$$

and the corresponding negative eigenvalue of the partial transpose is given by

$$\lambda_n = -\frac{\tanh^{2n} r}{2 \cosh^3 r} \sqrt{n+1}, \quad (34)$$

The degree of entanglement for the two observers here can be measured by using the the concept of logarithmic negativity [24]. The logarithmic negativity is defined as

$$E_{\mathcal{N}}(\rho) \equiv \log_2(2\mathcal{N}(\rho) + 1), \quad (35)$$

where $\mathcal{N}(\rho)$ is the negativity of the state. The negativity is defined as the absolute sum of the negative eigenvalues of the partial transpose with respect to ρ_{AB}^T . So

$$\mathcal{N}(\rho) \equiv \sum_n \frac{|\lambda_n| - \lambda_n}{2}. \quad (36)$$

The entanglement monotone is found to be

$$E_{\mathcal{N}}(\rho) = \log_2 \left(1 + \sum_{n=0}^{\infty} \frac{\tanh^{2n} r}{\cosh^3 r} \sqrt{n+1} \right). \quad (37)$$

At $r \rightarrow 0$, corresponding to $\kappa \rightarrow 0$, we are back to the usual case of entanglement between Alice and Bob where both are Minkowski observers, and there are no Schwarzschild modes in Bob's cavity (i.e. $n = 0$). And thus $E_{\mathcal{N}}(\rho) = 1$. For a finite acceleration the entanglement is degraded. In the limit $r \rightarrow \infty$, the second term in (37) is vanishing. The logarithmic negativity is then 0. Figure 2 shows that for a finite surface gravity, the degree of entanglement is reduced. Figure 3 shows that if the black hole mass M is fixed, the logarithmic negativity is reduced as the extra dimension number n increases. On the other hand, if n is fixed, $E_{\mathcal{N}}$ is enhanced as M increases.

The states $|0\rangle_M, |1\rangle_M$ are defined in terms of the physical Fock states for Alice's cavity by the dual-rail basis states as suggested by Ref. [6]: $|0\rangle_M = |1\rangle_{A_1} |0\rangle_{A_2}$, $|1\rangle_M = |0\rangle_{A_1} |1\rangle_{A_2}$, and the similar expressions for Bob's cavity. In order to teleport the unknown state $|\varphi\rangle_M = \alpha|0\rangle_M + \beta|1\rangle_M$ to Bob, we should assume that Alice has an additional cavity, which contains a single qubit with dual-rail encoding by a photon excitation of a two-mode incoming (Minkowski) vacuum state. This will allow Alice to perform a joint measurement on the two orthogonal modes of each cavity. After Alice's measurement, Bob's photon will be projected according to the measurement outcome. The final state Bob received can be given by $|\varphi_{ij}\rangle = x_{ij}|0\rangle + y_{ij}|1\rangle$, where there are four possible conditional state amplitudes expressed as $(x_{00}, y_{00}) = (\alpha, \beta)$, $(x_{01}, y_{01}) = (\beta, \alpha)$, $(x_{10}, y_{10}) = (\alpha, -\beta)$, and $(x_{11}, y_{11}) = (-\beta, \alpha)$. Once receiving Alice's results of measurement, Bob can apply a unitary transformation to verify the protocol in his local frame. However, Bob must be confronted with the fact that his cavity will become teemed with thermally excited photons because of the Hawking effect.

When Alice sends the result of her measurement to Bob, the state he observes must be traced out over region II ,

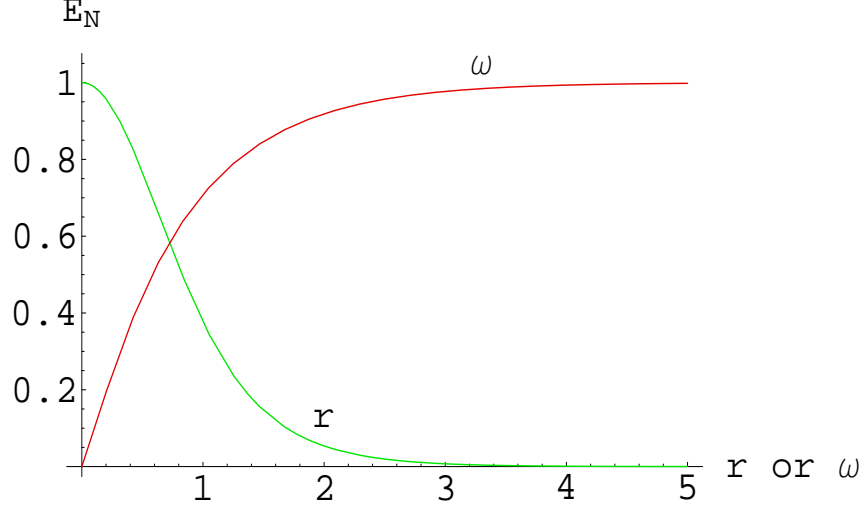


FIG. 2: The logarithmic negativity as a function of r ($e^{2r} = \coth(\pi\omega/2\kappa)$), and the mode frequency ω . The green line shows that the entanglement monotone reduces with r when ω is fixed, while the red line shows that for the fixed surface gravity the entanglement monotone increases with ω .

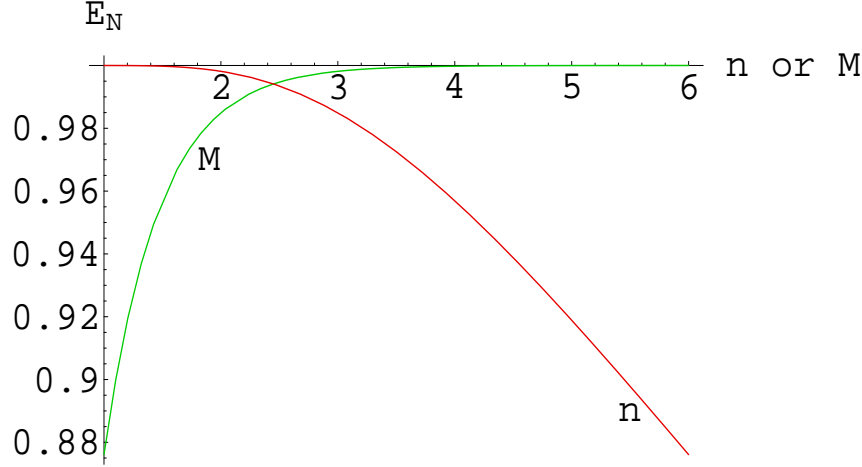


FIG. 3: The logarithmic negativity as a function of the extra dimensions ($n = d - 4$) and the black hole mass M .

since Rob is causally disconnected from region II [6],

$$\begin{aligned}
 \rho_{ij}^{(I)} &= Tr_{II}(|\varphi_{ij}\rangle_M \langle \varphi_{ij}|) = \sum_{n=0}^{\infty} p_n \rho_{ij,n}^I \\
 &= \frac{1}{\cosh^6 r} \sum_{n=0}^{\infty} \sum_{m=0}^n [(\tanh^2 r)^{n-1} [(n-m)|x_{ij}|^2 + m|y_{ij}|^2] \\
 &\quad \times |m, n-m\rangle_I \langle m, n-m| + (x_{ij} y_{ij}^* \tanh^{2n} r \\
 &\quad \sqrt{(m+1)(n-m+1)}) \times |m, n-m+1\rangle_I \\
 &\quad \langle m+1, n-m| + \text{H.c.}] ,
 \end{aligned} \tag{38}$$

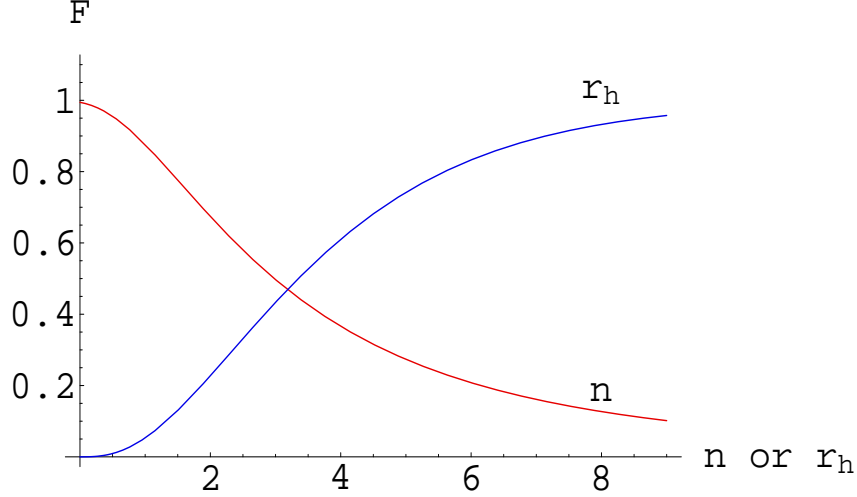


FIG. 4: The fidelity as the function of the extra dimensions n and horizon radius r_h . The fidelity decreases with the extra dimensions, while the fidelity increases with the horizon radius.

in particular with

$$p_0 = 0, \quad p_1 = 1/\cosh^6 r, \quad p_n = \frac{(\tanh^2 r)^{n-1}}{\cosh^6 r}. \quad (39)$$

Since what we are concerned is to which extent $|\varphi_{ij}\rangle$ might deviate from unitarity, it is reasonable for us to perform a unitary transformation on $|\varphi_{ij}\rangle$ and convert its form into $|\varphi\rangle_I$. Suppose upon receiving the result (i, j) of Alice's measurement, Bob can apply the rotation operators restricted to the 1-excitation sector of his state. In this way, we may define the fidelity corresponding to the teleportation as

$$\begin{aligned} F^I(|\varphi\rangle) &= {}_I\langle\varphi|\rho^I\hat{U}|\varphi_{ij}\rangle_I \\ &= {}_I\langle\varphi|\rho^I|\varphi\rangle_I = (1 - e^{-\pi\omega/\kappa})^3, \end{aligned} \quad (40)$$

which is identical with the results of Alsing and Milburn. From Fig. 4, we can see that the fidelity of teleportation depends on the extra dimension number and horizon radius.

We notice that the Hawking radiation emitted by larger four dimensional, astrophysical black holes has not yet been observed because for a black hole with several solar mass M_\odot , the characteristic temperature is about $T_H \sim 10^{-8}\text{K}$: an extremely low temperature corresponding to a very low energy frequency that cannot be detected. The primordial black holes that would have been created at the early universe with possible mass ($M_{BH} \sim 10^{15}\text{gr}$) and the corresponding Hawking spectrum with a peak in the range 10 – 100 MeV have not been detected either.

In recent years, it is proposed that in brane-world theories the true scale M_* of quantum gravity may be lower than the traditional Planck scale M_{pl} , possibly approaching TeV scales [12]. The observed weakness of gravity at long distances is due to the presence of n new spatial dimensions large compared to the electroweak scale. This follows that $M_{pl}^2 \sim R^n M_*^{n+2}$, where R is the size of the extra dimensions. A black hole with horizon radius r_h smaller than the size of extra dimensions R , would submerge into extra dimensions, and for a given mass M such a black hole becomes lighter, larger and colder than a usual four dimensional black hole with the same mass. If we assume that $M_* = 1\text{ TeV}$, which implies that $R \sim 10^{30/n-17}\text{cm}$ and, choose the black hole mass $M_{BH} = 5\text{ TeV}$, the ratio between the $(4+n)$ -dimensional Schwarzschild horizon radius and the ordinary 4-dimensional Schwarzschild with the same mass $M_{BH} = 5\text{ TeV}$ is given by [25]

$$\frac{r_{h(4)}}{r_{h(4+n)}} \sim \left(\frac{r_{h(4+n)}}{R}\right)^n. \quad (41)$$

For $n = 2$, we have $r_{h(4+n)} = 2.6 \times 10^{-19}\text{ m}$ and then $r_{h(4)}/r_{h(4+n)} \sim 10^{-30}$. So for a given mode frequency ω , the TeV-level gravity enhances the entanglement monotone and teleportation fidelity greatly. But for a mini-black hole that may be created at LHC with emitted Hawking radiation frequency range $0.05 \lesssim \omega r_h \lesssim 0.5$, the logarithmic negativity for entanglement is then bounded to $E_{\mathcal{N}_{\max}} = 0.77$ and the teleportation saturates to $F_{\max} = 0.25$.

III. HIGHER DIMENSIONAL ROTATING BLACK HOLE CASES

In this section, we discuss entanglement and teleportation in the background spacetime of a higher dimensional rotating black hole. The spacetime around a $(4+n)$ -dimensional, rotating, uncharged black hole is given by the following line-element [22]

$$ds^2 = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2 \theta}{\Sigma r^{n-1}} dt d\varphi - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \left(r^2 + a^2 + \frac{a^2 \mu \sin^2 \theta}{\Sigma r^{n-1}}\right) \sin^2 \theta d\varphi^2 - r^2 \cos^2 \theta d\Omega_n, \quad (42)$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \quad \Sigma = r^2 + a^2 \cos^2 \theta, \quad (43)$$

and $d\Omega_n$ is the line-element on a unit n -sphere. The mass and the angular momentum per unit mass of the black hole are given by,

$$M = \frac{(n+2)A_{n+2}}{16\pi G_{4+n}}\mu, \quad \frac{J}{M} = \frac{2}{n+2}a, \quad (44)$$

with G_{4+n} being the $(4+n)$ -dimensional Newton's constant, and A_{n+2} the area of a $(n+2)$ -dimensional unit sphere given by

$$A_{n+2} = \frac{2\pi^{(n+3)/2}}{\Gamma[(n+3)/2]}. \quad (45)$$

The horizon occurs when $\Delta(r) = 0$, i.e., when $r = r_h$ with

$$r_h = \left[\frac{\mu}{1 + a_*} \right]^{1/(n+1)}, \quad (46)$$

where $a_* = a/r_h$. Note that there is only a single horizon when $n \geq 1$, in contrast to the four-dimensional Kerr black hole, which has an inner and an outer horizon. We consider μ and a as the normalized mass and angular momentum parameters, respectively. Also note that there is no upper bound on a when $n \geq 2$, in contrast to the four dimensional case when a is bounded. The surface gravity and the angular velocity at the horizon are given by [26]

$$\kappa = \frac{(n+1) + (n-1)a_*^2}{2(1+a_*^2)r_h}, \quad \Omega = \frac{a_*}{(1+a_*^2)r_h}. \quad (47)$$

Greybody factors for five-dimensional rotating black holes on the brane were calculated in Ref. [26].

In this section, we will investigate quantum entanglement and teleportation not only with scalar particles, but also with gauge bosons and Dirac particles in the spacetime of a higher-dimensional rotating black hole. The angular momentum parameter a (or a_*) will definitely affect the degree of entanglement and fidelity of teleportation. In order to derive a master equation describing the motion of a field with arbitrary spin s , we need to make use of the Newman-Penrose formalism [27, 28]. The master equations for a spin s field, under the standard decomposition

$$\phi = R_s(r)S(\theta)e^{-i\omega t + im\varphi}, \quad (48)$$

read as [26, 29]

$$\Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{dR_s}{dr} \right) + \left[\frac{K^2 - i[2r + (n-1)\mu r^{-n}]K}{\Delta} + 4is\omega r + s(\Delta'' - 2) - \Lambda_{sj} \right] R_s = 0, \quad (49)$$

$$\frac{1}{\sin \theta} \frac{d}{d\sin \theta} \left(\sin \theta \frac{dS}{d\theta} \right) + \left[-\frac{2ms \cot \theta}{\sin \theta} - \frac{m^2}{\sin^2 \theta} + a^2 \omega^2 \cos^2 \theta - 2as\omega \cos \theta + s - s^2 \cot^2 \theta + \lambda_{sj} \right] S = 0, \quad (50)$$

where

$$\begin{aligned} K &= (r^2 + a^2)\omega - am, \quad \Lambda_{sj} = \lambda_{sj} + a^2\omega^2 - 2am\omega, \\ \text{and } \lambda_{sj} &= j(j+1) - s(s+1) - \frac{2ms^2}{j(j+1)}a\omega + \dots \end{aligned} \quad (51)$$

The asymptotic solutions of Eq. (48) coming out the horizon are given by

$$\phi_I = e^{-i(\omega-\omega_0)(\hat{t}-r_*)}, \quad (52)$$

$$\phi_{II} = e^{-i(\omega-\omega_0)(r_*-\hat{t})}, \quad (53)$$

where

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}, \quad \omega_0 = m\Omega, \quad \text{and} \quad \hat{t} = \frac{\omega}{\omega - \omega_0} t \quad (54)$$

where r_* is the tortoise coordinate. By defining the generalized light-like Kruskal coordinates [30]

$$U = -\frac{1}{\kappa} e^{-\kappa u}, \quad V = \frac{1}{\kappa} e^{\kappa v}, \quad \text{for } r > r_h, \quad (55)$$

$$U = \frac{1}{\kappa} e^{-\kappa u}, \quad V = \frac{1}{\kappa} e^{\kappa v}, \quad \text{for } r < r_h, \quad (56)$$

$$u = \hat{t} - r_*, \quad v = \hat{t} + r_* \quad (57)$$

we can rewrite (52) and (53) in the following form,

$$\phi_I = \exp\left[\frac{i(\omega - \omega_0)}{\kappa} \ln(-\kappa U)\right], \quad (58)$$

$$\phi_{II} = \exp\left[\frac{i(\omega - \omega_0)}{\kappa} \ln(\kappa U)\right]. \quad (59)$$

By using the formula $-1 = e^{i\pi}$ and making (58) analytic in the lower half-plane of U , we find a complete basis for positive energy U modes

$$\phi_1 = (-U/a)^{\frac{i(\omega-\omega_0)}{\kappa}} + e^{\frac{\pi(\omega-\omega_0)}{\kappa}} (U/a)^{\frac{i(\omega-\omega_0)}{\kappa}}, \quad (60)$$

$$\phi_2 = (-U/a)^{\frac{-i(\omega-\omega_0)}{\kappa}} + e^{\frac{-\pi(\omega-\omega_0)}{\kappa}} (U/a)^{\frac{-i(\omega-\omega_0)}{\kappa}}. \quad (61)$$

The Hawing radiation spectrum can be obtained by using $(N_\omega \phi_1, N_\omega \phi_1) = N_\omega^2 (1 \pm e^{\frac{2\pi(\omega-\omega_0)}{\kappa}}) = -1$, i.e.,

$$N_\omega^2 = \frac{1}{e^{\frac{2\pi(\omega-\omega_0)}{\kappa}} \pm 1}, \quad (62)$$

where $+$ corresponds to fermions and $-$ bosons. Similarly to the Schwarzschild case, the vacuum in scalar and Maxwell fields can also be written in the two-mode squeezed state,

$$|0\rangle_M = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n\rangle_I \otimes |n\rangle_{II}, \quad (63)$$

where $\cosh r = (1 - e^{-2\pi(\omega-\omega_0)/\kappa})^{-1/2}$ and $\tanh r = e^{\pi(\omega-\omega_0)/\kappa}$.

For Dirac fields, the Bogoliubov transformations between the Kruskal and Schwarzschild operators are given by [?]]

$$\begin{aligned} a_p^I &= \cos r c_p^I + \sin r c_p^{II\dagger}, \\ a_p^{II} &= \cos r c_p^{II} + \sin r c_p^{I\dagger}, \end{aligned} \quad (64)$$

where the fermionic Bogoliubov coefficients allow us to define

$$\cos r = (1 + e^{-2\pi(\omega-\omega_0)/\kappa})^{-1/2}, \quad \sin r = e^{-\pi(\omega-\omega_0)/\kappa} (1 + e^{-2\pi(\omega-\omega_0)/\kappa})^{-1/2}. \quad (65)$$

As for the scalar field case, the Bogoliubov transformation can also be expressed as

$$a_p^{\sigma\dagger} = \tilde{S}_p(r) c_p^{\sigma\dagger} \tilde{S}_p^\dagger(r), \quad c_p^{\sigma\dagger} = \tilde{T}_p(r) a_p^{\sigma\dagger} \tilde{T}_p^\dagger(r), \quad (\sigma = I, II) \quad (66)$$

in terms of the two-mode squeeze operators

$$\tilde{S}_p(r) = \exp[-r(c_p^I c_p^{II} - c_p^{I\dagger} c_p^{II\dagger})], \quad \tilde{T}_p(r) = \exp[r(a_p^I a_p^{II} - a_p^{I\dagger} a_p^{II\dagger})] \quad (67)$$

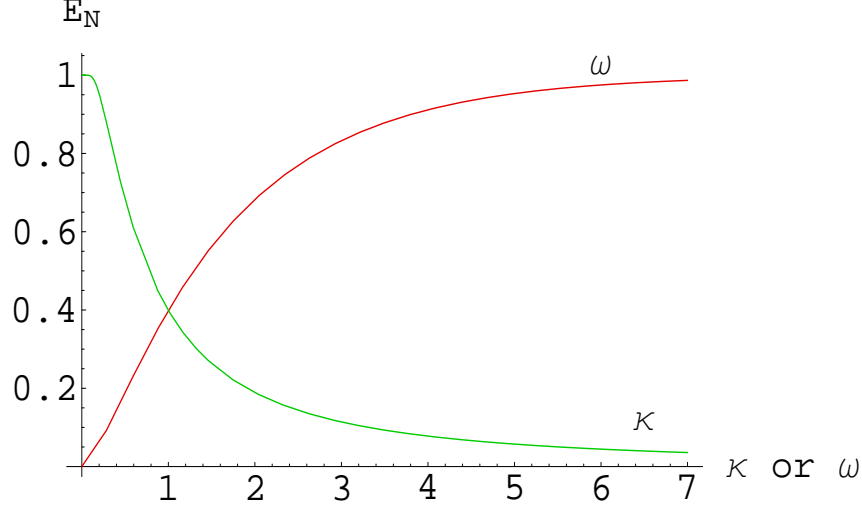


FIG. 5: The logarithmic negativity as a function of the surface gravity κ and the mode frequency ω .

Then the Dirac vacuum defined by $a_p^\sigma |0\rangle_M = 0$ is given by

$$\begin{aligned} |0\rangle_M &= \tilde{T}_p(r) |0\rangle_I \otimes |0\rangle_{II} \\ &= e^{\ln \cos r} \exp[\ln(1 + \tan r c_p^{I\dagger} c_{\bar{p}}^{II\dagger})] |0\rangle_I \otimes |0\rangle_{II}. \end{aligned} \quad (68)$$

Here $N_f = \cos r$ is the fermionic normalization factor. Considering the finite number of allowed excitations in the fermionic system due to the Pauli exclusion principle, Eq. (68) can be rewritten as

$$|0\rangle_M = N_f \{ |0\rangle_I \otimes |0\rangle_{II} + \sum_p \tan r (|1_p\rangle_I \otimes |1_{\bar{p}}\rangle_{II} + |1_{\bar{p}}\rangle_I \otimes |1_p\rangle_{II}) \}. \quad (69)$$

Since we only need to consider the information teleported out of the black hole, we can drop the second set of parentheses in Eq. (68). We now simplify our analysis by considering the effect of teleportation of the initial quantum state inside the horizon to a single mode of the outside Hawking particles, which goes as

$$|0\rangle_M = \cos r |0\rangle_I \otimes |0\rangle_{II} + \sin r |1\rangle_I \otimes |1\rangle_{II}. \quad (70)$$

A. Quantum entanglement and teleportation with scalar particles and gauge bosons

In this subsection, we discuss quantum entanglement and teleportation with scalar particles and gauge bosons in the rotating black hole spacetime. The form formula for entanglement monotone is the same as that of the Schwarzschild case, except for the difference that now the angular momentum parameter contributes to the value of the surface gravity. Thus, the logarithmic negativity as a function of the angular momentum parameter a (or a_*) is written as

$$E_N(\rho) = \log_2 \left(1 + \sum_{n=0}^{\infty} \frac{\tanh^{2n} r}{\cosh^3 r} \sqrt{n+1} \right). \quad (71)$$

The analytic properties of this entanglement monotone are shown in Fig. 5 and Fig. 6. Figure 5 demonstrates that when the surface gravity increases and the mode frequency ω is fixed, the logarithmic negativity decreases. But if we fix the surface gravity κ and change the frequency ω , the logarithmic negativity increases with ω .

Figure 6 shows that if we fix the angular momentum parameter a and vary the mass μ (or the horizon radius), the logarithmic negativity increases as the mass (or the horizon radius) increases. If we fix the black hole mass and vary the angular momentum parameter a , the logarithmic negativity increases with a_* , but the correction of entanglement monotone is small due to the different value of a_* . This is different from a 4-dimensional Kerr black hole [11]. On

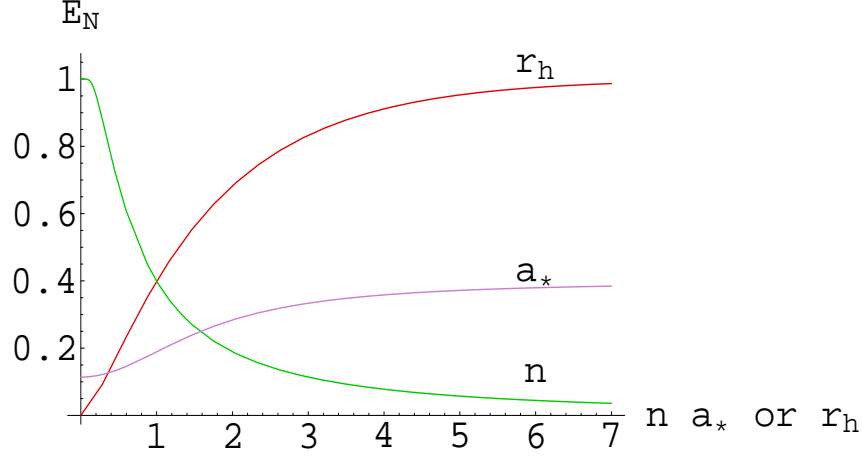


FIG. 6: The logarithmic negativity as a function of the horizon radius r_h , the angular momentum a_* , and the extra dimensions n .

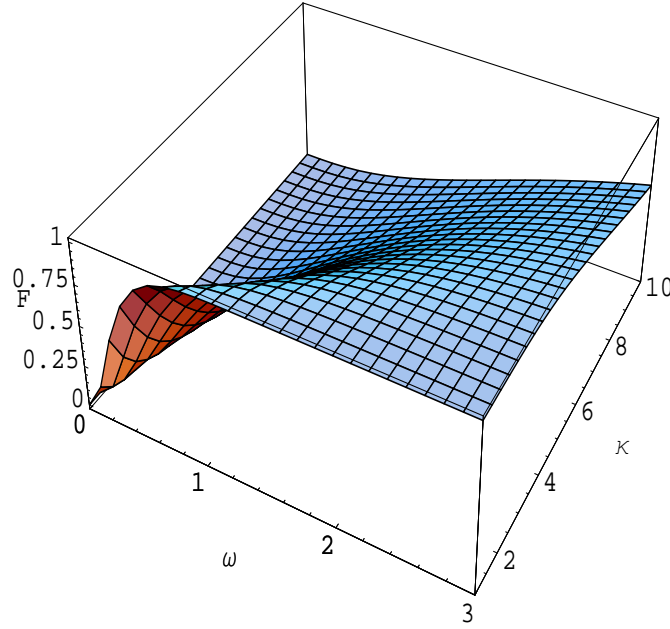


FIG. 7: The fidelity of teleportation with bosonic particles for a $(4+n)$ -dimensional Kerr black hole as a function of the mode frequency ω and surface gravity κ .

the other hand, if we keep the mass and the angular momentum parameter a_* fixed, but vary the extra dimensions of spacetime, the logarithmic negativity is reduced as n increases.

The quantum teleportation fidelity between Alice and Bob can also be described by Eq. (40), that is to say,

$$F^I(|\varphi\rangle) = (1 - e^{-\pi(\omega - \omega_0)/\kappa})^3. \quad (72)$$

Figure 7 shows that the fidelity of teleportation is closely related to the mode frequency and the surface gravity. Increasing the surface gravity reduces the value of fidelity, while the mode frequency does not.

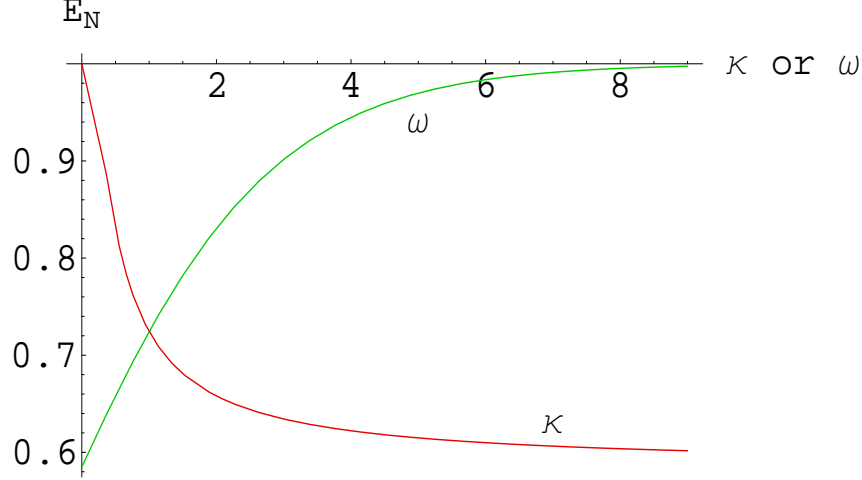


FIG. 8: Logarithmic negativity as a function of the surface gravity κ and the mode frequency ω . Note that here $E_{\mathcal{N}\min} = 0.58$

B. Quantum entanglement and teleportation with Dirac particles

It is interesting to study quantum entanglement and teleportation with fermions, since the Pauli exclusion principle becomes important in this case. The 1-excitation Minkowski Fock state can be obtained by acting the operator $a_M^\dagger = \cos r c_I^\dagger + \sin r c_{II}^\dagger$ on the Minkowski vacuum $|0\rangle_M$, which reads as

$$|1\rangle_M^{\mathcal{B}} = |1\rangle_I \otimes |0\rangle_{II}. \quad (73)$$

Thus, we rewrite Eq. (32) in terms of Minkowski modes for Alice and Kerr modes for Bob. We trace over region II and obtain the resulting state

$$\begin{aligned} \rho_{AB} = \frac{1}{2} & (\cos^2 r |00\rangle\langle 00| + \sin^2 r |01\rangle\langle 01| \\ & + \cos r |00\rangle\langle 11| + \cos r |11\rangle\langle 00| + |11\rangle\langle 11|). \end{aligned} \quad (74)$$

The partial transpose for ρ_{AB} is obtained by interchanging Alice's qubit as

$$\rho_{AB}^T = \frac{1}{2} \begin{pmatrix} \cos^2 r & 0 & 0 & 0 \\ 0 & \sin^2 r & \cos r & 0 \\ 0 & \cos r & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

which yields one negative eigenvalue

$$\lambda_1 = -\cos^2 r / 2. \quad (75)$$

The logarithmic negativity is found to be

$$E_{\mathcal{N}} = \log_2 (1 + \cos^2 r) \quad (76)$$

From Eq. (76), we can see that when the surface gravity κ approaches infinity, the logarithmic negativity saturates to $\lim_{\kappa \rightarrow \infty} E_{\mathcal{N}} \simeq 0.58$ (see Fig. 8).

We now turn to teleportation between Alice and Bob and utilize dual rail basis states as an excitation of a spin-up state in one of two possible spatial modes in Alice's cavity and similarly for Bob. Upon receiving Alice measurement results, Bob trace over region II of the state he observes and obtains [6],

$$\begin{aligned} \rho^I &= \sum_{k=0}^1 \sum_{l=0}^1 I \langle k, l | \phi_{ij} \rangle \langle \phi_{ij} | k, l \rangle_I \\ &= \cos^2 r |\phi_{ij}\rangle \langle \phi_{ij}| + \sin^2 r |11\rangle_I \langle 11|. \end{aligned} \quad (77)$$

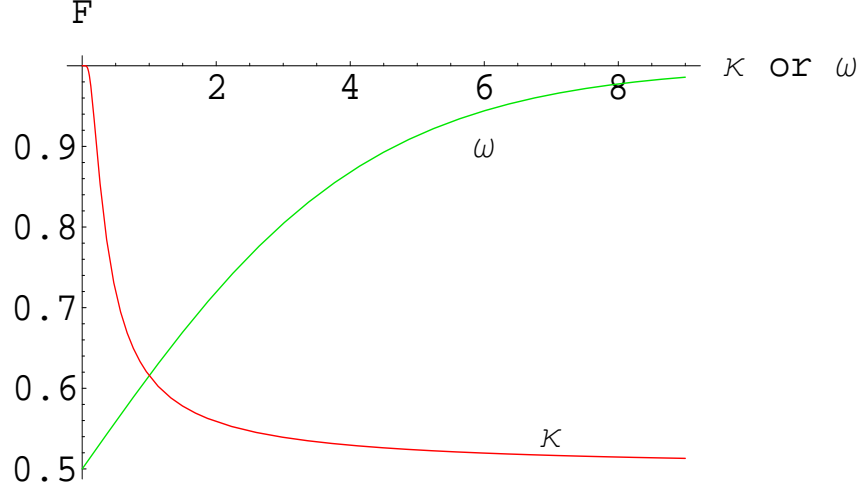


FIG. 9: The fidelity of teleportation is degraded with the surface gravity κ but is bounded to $1/2$ as $\kappa \rightarrow \infty$.

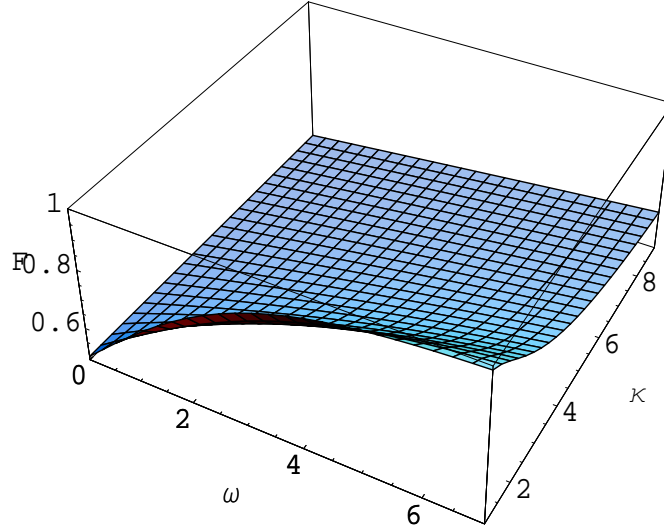


FIG. 10: The fidelity of teleportation with Dirac particles as a function of the surface gravity and mode frequency.

The fidelity of Bob's final state is then

$$F^I = \text{Tr}_I (|\psi\rangle\langle\psi|\rho^I) = \cos^2 r. \quad (78)$$

In the fermionic case, the number of allowed excitations is bounded above by $n = 2$. Thus when the acceleration approaches infinity, the fidelity saturates to $\lim_{\kappa \rightarrow \infty} \cos^2 r = 1/2$ (see Fig.9). Figure 10 demonstrates the fidelity of teleportation as a function of both the mode frequency and black hole surface gravity. For a mini-black hole that may be created at the LHC with emitted Hawking radiation frequency range $0.05 \lesssim \omega r_h \lesssim 0.5$, the logarithmic negativity for entanglement is then bounded to $E_{\mathcal{N}_{\max}} = 0.73$ and the teleportation saturates to $F_{\max} = 0.65$.

IV. DISCUSSIONS AND CONCLUSIONS

In summary, we have investigated quantum entanglement and teleportation in the spacetime of higher dimensional Schwarzschild and Kerr black holes. The teleportation protocols in the black hole spacetime are discussed in detail. For massive black holes and well-insulated cavities, which can reflect all non-gravitational fields, it is possible for an observer with his cavity to lower the cavity slowly down to the black hole horizon, and perfect entanglement and teleportation can be realized since the gravitational coupling is negligible. However, for mini-black holes, we prefer to let one observer (Bob) fall freely, then slow down and stop to become stationary at a fixed radius. Actually, it is hard to believe that we can keep a thick wall cavity near a mini-black hole horizon in thermal equilibrium with the Hawking radiation. If we want the the cavity to be in thermal equilibrium, the wall of the cavity should be very thin (compared to the size of the horizon curvature) and a thin wall cavity could not prevent outside non-gravitational fields from leaking into the cavity. We have discussed a possible new physics that may arise from the presence of extra dimensions and the unbounded angular momentum parameter a_* . It is found that if the gravity is really at TeV scales, the reduction of entanglement monotone and teleportation fidelity is not so drastic as that in Planck scale gravity theory for mini-black holes. We can use quantum entanglement and quantum teleportation experiments to detect extra dimensions. For instance, for a mini-black hole created at LHC, quantum entanglement experiments can be conducted near this black hole. If this black hole is a higher dimensional one, the reduction of entanglement monotone is not so drastic as that of an ordinary 4-dimensional one. The reduction of entanglement monotone can be described as a function of extra dimensions. If we fix other parameters, then the degree of entanglement decreases as the number of extra dimensions increases.

The degree of entanglement is found to be degraded with increasing the extra dimensions. For a finite black hole surface gravity, the observer may choose higher frequency mode to keep high level entanglement. The fidelity of quantum teleportation is also reduced because of the Hawking-Unruh effect. We have discussed the fidelity as a function of the extra dimensions, mode frequency, mass and/or angular momentum parameter of black hole for both bosonic and fermionic resources. For quantum entanglement and teleportation with bosonic particles, all the correlations might be lost for the observers in gravitational fields and the fidelity could be degraded to zero, but for fermionic cases, the entanglement monotone and teleportation fidelity are bounded to 0.58 and 0.5, respectively, because of the Pauli exclusion principle.

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